Octonionic electromagnetic and gravitational interactions and dark matter

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Based on the Maxwellian quaternionic electromagnetic theory, the electromagnetic interaction, the gravitational interaction and their coupling influence with the dark matter field in the octonionic space are discussed. The paper reveals the close relationships of dark matter field with electromagnetic field and gravitational field. In the dark matter field, the research discloses the movement properties of dark matter in certain conditions and its influence on ordinary matter movements. In the electromagnetic field, the variation of field energy density has direct effect on the force of electric charge and current. In the modified gravitational field, the near coplanarity, near circularity and corevolving of planetary orbits are deduced from the equations set. The change of centrifugal force of celestial body leads to fluctuation of revolution speed, when the field energy density, celestial body's angular momentum in its sense of revolution or its mass spatial distribution varied. The results explain that some observed abnormal phenomena about celestial bodies are caused by either modified gravitational field or dark matter.

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I. INTRODUCTION

Nowadays, there still exist some movement phenomena of celestial body which can't be explained by current gravitational theory. Therefore, some scientists doubt the universality of current gravitational theory, and then bring forward the modified gravitational theory and hypothesis of dark matter to explain the phenomena of abnormal rotation velocity of galaxy and the associated 'missing mass' etc. The hypothesis of dark matter believes that, there exist not only various gleamy ordinary matters, but also one kind of mysterious matter which exerts the gravitational pull, but neither emits nor absorbs light [1]. This kind of matter is known as dark matter, which owns mass and has effect on ordinary matter movements [2]. Most scientists believe in its existence and multiformity in the universe.

A new insight on the problem of the dark matter can be given by the concept of the octonion space. The understanding of time and space experiences an evolutional period [3]. In 17th century, I. Newton thought the time and space in nature were separate. And the time and space were both independent of matter. In 20th century, A. Einstein believed that, the time and space in nature were integrated and commonly known as the spacetime. The spacetime related to the movement status and matter. The precursors include R. Descartes and A. Einstein etc. held the idea that space and time were the extension of matter. And there was no such spacetime without matter and hence no empty spacetime [4].

In the paper, we believe that electromagnetic and gravitational interactions are different, equal and relative. And they possess different and four-dimensional spacetimes respectively. These two types of different and four-dimensional spacetimes are equal and independent, and can not be superposed directly. The viewpoint can be summarized as 'SpaceTime Equality Postulation', that is: each fundamental interaction possesses its own spacetime, and all these spacetimes are equal.

According to previous research results and the 'SpaceTime Equality Postulation', the electromagnetic and gravitational interactions can be described by their four-dimensional spacetimes. Based on the conception of space verticality etc., two types of four-dimensional spacetimes can be united into an eight-dimensional spacetime. In eight-dimensional spacetime, the electromagnetic interaction and gravitational interaction can be equally described. So the characteristics of electromagnetic and gravitational interactions can be described by a single eight-dimensional spacetime uniformly.

The paper modifies the gravitational theory of ordinary matter and dark matter, and draws some conclusions which are consistent with the Maxwellian electromagnetic theory and the Newtonian gravitational theory. A few prediction associated with movement feature of dark matter can be deduced, and some new particles can be used for the candidate of dark matter.

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II. ELECTROMAGNETIC FIELD AND QUATERNION SPACE

The electromagnetic theory can be described with the quaternion algebra [5]. In the treatise on electromagnetic theory, the quaternion algebra was first used by J. C. Maxwell to describe the equations set and various properties of the electromagnetic field [6]. The spacetime, which is associated with electromagnetic interaction and possesses the physics content, is adopted by the quaternion space.

In the quaternion space of electromagnetic interaction, the radius vector $\mathbb{R} = (r_0, r_1, r_2, r_3)$, and the base $\mathbb{E} = (1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3)$. Where, $r_0 = ct$, c is the speed of light beam, and t denotes the time.

The quaternion differential operator $\lozenge$ and its conjugate operator $\lozenge^*$ are defined as

$$\lozenge = \partial_0 + \boldsymbol{i}_1\partial_1 + \boldsymbol{i}_2\partial_2 + \boldsymbol{i}_3\partial_3 \ , \quad \lozenge^* = \partial_0 - \boldsymbol{i}_1\partial_1 - \boldsymbol{i}_2\partial_2 - \boldsymbol{i}_3\partial_3 \ . \tag{1}$$

where, $\partial_i = \partial/\partial r_i$, i = 0, 1, 2, 3 . The mark (*) represents the quaternion conjugate.

The electromagnetic field potential $\mathbb{A}(a_0, a_1, a_2, a_3)$ is defined as

$$\mathbb{A} = \lozenge \circ \mathbb{X} = a_0 + \boldsymbol{i}_1 a_1 + \boldsymbol{i}_2 a_2 + \boldsymbol{i}_3 a_3 \tag{2}$$

where, $\mathbb{X}$ is the physical quantity.

A. Electromagnetic field strength

The electromagnetic field strength $\mathbb{B}(b_0, b_1, b_2, b_3)$ is defined as

$$\begin{aligned} \mathbb{B} = & \ \lozenge \circ \mathbb{A} \\ = & \ b_{00} + (\boldsymbol{i}_1 b_{01} + \boldsymbol{i}_2 b_{02} + \boldsymbol{i}_3 b_{03}) + (\boldsymbol{i}_1 b_{32} + \boldsymbol{i}_2 b_{13} + \boldsymbol{i}_3 b_{21}) \\ = & \ (\partial_0 a_0 - \partial_1 a_1 - \partial_2 a_2 - \partial_3 a_3) \\ & + \{\boldsymbol{i}_1(\partial_0 a_1 + \partial_1 a_0) + \boldsymbol{i}_2(\partial_0 a_2 + \partial_2 a_0) + \boldsymbol{i}_3(\partial_0 a_3 + \partial_3 a_0)\} \\ & + \{\boldsymbol{i}_1(\partial_2 a_3 - \partial_3 a_2) + \boldsymbol{i}_2(\partial_3 a_1 - \partial_1 a_3) + \boldsymbol{i}_3(\partial_1 a_2 - \partial_2 a_1)\} \end{aligned} \tag{3}$$

where, the first term in the right of equal mark is gauge definition, the second term is electric field intensity (b_{01}, b_{02}, b_{03}) and the third term is flux density (b_{32}, b_{13}, b_{21}). Taking $b_0 = b_{00} = 0$, then the gauge condition of field potential in electromagnetic field can be achieved.

TABLE I: The quaternion multiplication table.

	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$
1	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$
$\boldsymbol{i}_1$	$\boldsymbol{i}_1$	-1	$\boldsymbol{i}_3$	$-\boldsymbol{i}_2$
$\boldsymbol{i}_2$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_3$	-1	$\boldsymbol{i}_1$
$\boldsymbol{i}_3$	$\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_1$	-1

B. Electromagnetic field source

The electromagnetic field source $\mathbb{S}(s_0, s_1, s_2, s_3)$ is defined as

$$\begin{aligned} -\mu\mathbb{S} = & \ \lozenge^* \circ \mathbb{B} \\ = & \ (\partial_0 b_0 + \partial_1 b_1 + \partial_2 b_2 + \partial_3 b_3) \\ & + \{\boldsymbol{i}_1(\partial_0 b_1 - \partial_1 b_0) + \boldsymbol{i}_2(\partial_0 b_2 - \partial_2 b_0) + \boldsymbol{i}_3(\partial_0 b_3 - \partial_3 b_0)\} \\ & + \{\boldsymbol{i}_1(\partial_3 b_2 - \partial_2 b_3) + \boldsymbol{i}_2(\partial_1 b_3 - \partial_3 b_1) + \boldsymbol{i}_3(\partial_2 b_1 - \partial_1 b_2)\} \\ = & \ \lozenge^2(a_0 + \boldsymbol{i}_1 a_1 + \boldsymbol{i}_2 a_2 + \boldsymbol{i}_3 a_3) \end{aligned} \tag{4}$$

where, $\lozenge^2 = \lozenge^* \circ \lozenge$. μ is a coefficient. $b_0 = 0$. Taking $\mathbb{S} = (s_0 + \boldsymbol{i}_1 s_1 + \boldsymbol{i}_2 s_2 + \boldsymbol{i}_3 s_3)$, then equations set of field potential of the electromagnetic field can be attained

$$\lozenge^2(a_0 + \boldsymbol{i}_1 a_1 + \boldsymbol{i}_2 a_2 + \boldsymbol{i}_3 a_3) = -\mu(s_0 + \boldsymbol{i}_1 s_1 + \boldsymbol{i}_2 s_2 + \boldsymbol{i}_3 s_3) \tag{5}$$

TABLE II: Equivalent representation of the electromagnetic field equation.

	traditional representation	equivalent representation
field potential	$\boldsymbol{A}$	$\boldsymbol{A}$
	φ	φ
field strength	$\boldsymbol{B} = \nabla \times \boldsymbol{A}$	$\boldsymbol{B} = \nabla \times \boldsymbol{A}$
	$\boldsymbol{E} = -(\nabla\varphi + \partial\boldsymbol{A}/\partial t)$	$\boldsymbol{E}' = \nabla\varphi + \partial\boldsymbol{A}/\partial t$
	$\boldsymbol{D} = \varepsilon\, \boldsymbol{E}$	$\boldsymbol{D}' = \varepsilon\, \boldsymbol{E}'$
	$\boldsymbol{H} = \boldsymbol{B}/\mu$	$\boldsymbol{H} = \boldsymbol{B}/\mu$
gauge condition	$\nabla\cdot\boldsymbol{A} + \partial(\varphi/c)/\partial(ct) = 0$	$\nabla\cdot\boldsymbol{A} - \partial(\varphi/c)/\partial(ct) = 0$
field source	$\nabla^2\varphi - \partial^2\varphi/\partial(ct)^2 = -\rho/\varepsilon$	$\nabla^2\varphi + \partial^2\varphi/\partial(ct)^2 = -\rho/\varepsilon$
	$\nabla^2\boldsymbol{A} - \partial^2\boldsymbol{A}/\partial(ct)^2 = -\mu\boldsymbol{j}$	$\nabla^2\boldsymbol{A} + \partial^2\boldsymbol{A}/\partial(ct)^2 = -\mu\boldsymbol{j}$
Maxwell equation	$\nabla \times \boldsymbol{H} = \partial\boldsymbol{D}/\partial t + \boldsymbol{j}$	$\nabla \times \boldsymbol{H} = \partial\boldsymbol{D}'/\partial t + \boldsymbol{j}$
	$\nabla \times \boldsymbol{E} = -\partial\boldsymbol{B}/\partial t$	$\nabla \times \boldsymbol{E}' = \partial\boldsymbol{B}/\partial t$
	$\nabla \cdot \boldsymbol{D} = \rho$	$\nabla \cdot \boldsymbol{D}' = -\rho$
	$\nabla \cdot \boldsymbol{B} = 0$	$\nabla \cdot \boldsymbol{B} = 0$

C. Variation rate of field source

The variation rate $\mathbb{J}(j_0, j_1, j_2, j_3)$ of electromagnetic field source is defined as

$$\begin{aligned}
\mathbb{J} = &\ \lozenge^* \circ \mathbb{S} \\
= &\ (\partial_0 s_0 + \partial_1 s_1 + \partial_2 s_2 + \partial_3 s_3) \\
&+ \{\boldsymbol{i}_1(\partial_0 s_1 - \partial_1 s_0) + \boldsymbol{i}_2(\partial_0 s_2 - \partial_2 s_0) + \boldsymbol{i}_3(\partial_0 s_3 - \partial_3 s_0)\} \\
&+ \{\boldsymbol{i}_1(\partial_3 s_2 - \partial_2 s_3) + \boldsymbol{i}_2(\partial_1 s_3 - \partial_3 s_1) + \boldsymbol{i}_3(\partial_2 s_1 - \partial_1 s_2)\}
\end{aligned} \tag{6}$$

Taking the gauge condition $j_0 = 0$, then the continuity equation of field source of the electromagnetic field can be achieved.

In the quaternion space, various properties among the field potential, field strength and field source of the electromagnetic interaction can be described by the quaternion. Those conclusion and results are similar to that of equivalent representation of the electromagnetic field in Table 2. According to the 'SpaceTime Equality Postulation', it can be deduced that the spacetime derived from gravitational interaction is supposed to be the quaternion space also. Given that the coupling influence of fundamental interactions is neglected, the gravitational interaction is similar to electromagnetic interaction and can also be described by quaternion.

III. ELECTROMAGNETIC-GRAVITATIONAL FIELD AND OCTONION SPACE

The electromagnetic and gravitational interactions are interconnected, unified and equal. Both of them can be described in the quaternion space. By means of the conception of space expansion etc., two types of quaternion spaces can combine into an octonion space [7]. In the octonion space, various characteristics of electromagnetic and gravitational interactions can be described uniformly, and some correlative equations set can be obtained.

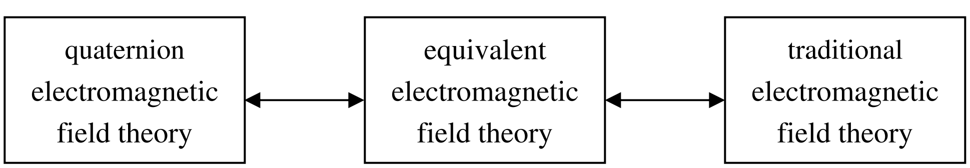

FIG. 1: Equivalent description in the electromagnetic field theory. In the quaternion space, the various properties among field potential, field strength and field source of electromagnetic interaction can be described by quaternion. Those conclusion and results are similar to that of equivalent representation of the electromagnetic field.

The base $\mathbb{E}_g = (1,\ \boldsymbol{i}_1,\ \boldsymbol{i}_2,\ \boldsymbol{i}_3)$ of quaternion space of gravitational interaction (G space, for short) is independent of the base $\mathbb{E}_e$ of quaternion space of electromagnetic interaction (E space, for short). Selecting $\mathbb{E}_e = (1,\ \boldsymbol{i}_1,\ \boldsymbol{i}_2,\ \boldsymbol{i}_3) \circ \boldsymbol{I}_0 = (\boldsymbol{I}_0,\ \boldsymbol{I}_1,\ \boldsymbol{I}_2,\ \boldsymbol{I}_3)$. And then the bases $\mathbb{E}_g$ and $\mathbb{E}_e$ constitute a single base $\mathbb{E}$ of the octonion space.

$$\mathbb{E} = \mathbb{E}_g + \mathbb{E}_e = (1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3, \boldsymbol{I}_0, \boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3) \tag{7}$$

The radius vector $\mathbb{R}(r_0, r_1, r_2, r_3, R_0, R_1, R_2, R_3)$ in the octonion space is

$$\mathbb{R} = (r_0 + \boldsymbol{i}_1 r_1 + \boldsymbol{i}_2 r_2 + \boldsymbol{i}_3 r_3) + (\boldsymbol{I}_0 R_0 + \boldsymbol{I}_1 R_1 + \boldsymbol{I}_2 R_2 + \boldsymbol{I}_3 R_3) \tag{8}$$

where, $r_0 = v_0 t$, $R_0 = V_0 T$. $v_0 = V_0 = c$ is the speed of light beam, t and T denote the time.

The octonion algebra is the alternative algebra, so the octonions $\mathbb{Q}_1$ and $\mathbb{Q}_2$ satisfy

$$\mathbb{Q}_1 \circ (\mathbb{Q}_1 \circ \mathbb{Q}_2) = (\mathbb{Q}_1 \circ \mathbb{Q}_1) \circ \mathbb{Q}_2\ , \quad \mathbb{Q}_1 \circ (\mathbb{Q}_2 \circ \mathbb{Q}_2) = (\mathbb{Q}_1 \circ \mathbb{Q}_2) \circ \mathbb{Q}_2\ .$$

The octonion differential operator $\lozenge$ and its conjugate operator $\lozenge^*$ are defined as

$$\lozenge = \lozenge_g + \lozenge_e\ , \quad \lozenge^* = \lozenge_g^* + \lozenge_e^*\ ; \tag{9a}$$

$$\lozenge_g = \partial_{g0} + \boldsymbol{i}_1 \partial_{g1} + \boldsymbol{i}_2 \partial_{g2} + \boldsymbol{i}_3 \partial_{g3}\ , \quad \lozenge_g^* = \partial_{g0} - \boldsymbol{i}_1 \partial_{g1} - \boldsymbol{i}_2 \partial_{g2} - \boldsymbol{i}_3 \partial_{g3}\ ; \tag{9b}$$

$$\lozenge_e = \boldsymbol{I}_0 \partial_{e0} + \boldsymbol{I}_1 \partial_{e1} + \boldsymbol{I}_2 \partial_{e2} + \boldsymbol{I}_3 \partial_{e3}\ , \quad \lozenge_e^* = -\boldsymbol{I}_0 \partial_{e0} - \boldsymbol{I}_1 \partial_{e1} - \boldsymbol{I}_2 \partial_{e2} - \boldsymbol{I}_3 \partial_{e3}\ . \tag{9c}$$

where, $\partial_{gi} = \partial / \partial r_i$; $\partial_{ei} = \partial / \partial R_i$; i = 0, 1, 2, 3 . The mark (*) represents the octonion conjugate.

The field potential $\mathbb{A}(a_0, a_1, a_2, a_3, A_0, A_1, A_2, A_3)$ in the electromagnetic-gravitational field is defined as

$$\mathbb{A} = \lozenge \circ \mathbb{X} = (a_0 + \boldsymbol{i}_1 a_1 + \boldsymbol{i}_2 a_2 + \boldsymbol{i}_3 a_3) + k_a (\boldsymbol{I}_0 A_0 + \boldsymbol{I}_1 A_1 + \boldsymbol{I}_2 A_2 + \boldsymbol{I}_3 A_3) \tag{10}$$

where, $\mathbb{X} = \mathbb{X}_g + k_x \mathbb{X}_e$; $\mathbb{X}_g$ and $\mathbb{X}_e$ are the physical quantity in G space and E space respectively; $\mathbb{A} = \mathbb{A}_g + k_a \mathbb{A}_e$; $\mathbb{A}_g = (a_0, a_1, a_2, a_3)$ and $\mathbb{A}_e = (A_0, A_1, A_2, A_3)$ are the field potential in G space and E space respectively; k_a and k_x are coefficients.

A. Field strength

The field strength $\mathbb{B}(b_0, b_1, b_2, b_3, B_0, B_1, B_2, B_3)$ is defined as

$$\begin{aligned}
\mathbb{B} = & \ \lozenge \circ \mathbb{A} \\
= & \ (b_0 + \boldsymbol{i}_1 b_1 + \boldsymbol{i}_2 b_2 + \boldsymbol{i}_3 b_3) + k_b (\boldsymbol{I}_0 B_0 + \boldsymbol{I}_1 B_1 + \boldsymbol{I}_2 B_2 + \boldsymbol{I}_3 B_3) \\
= & \ [(\partial_{g0} a_0 - \partial_{g1} a_1 - \partial_{g2} a_2 - \partial_{g3} a_3) \\
& + \boldsymbol{i}_1 (\partial_{g0} a_1 + \partial_{g1} a_0) + \boldsymbol{i}_2 (\partial_{g0} a_2 + \partial_{g2} a_0) + \boldsymbol{i}_3 (\partial_{g0} a_3 + \partial_{g3} a_0) \\
& + \boldsymbol{i}_1 (\partial_{g2} a_3 - \partial_{g3} a_2) + \boldsymbol{i}_2 (\partial_{g3} a_1 - \partial_{g1} a_3) + \boldsymbol{i}_3 (\partial_{g1} a_2 - \partial_{g2} a_1)] \\
& + [\boldsymbol{I}_0 (\partial_{e0} a_0 + \partial_{e1} a_1 + \partial_{e2} a_2 + \partial_{e3} a_3) \\
& + \boldsymbol{I}_1 (\partial_{e1} a_0 - \partial_{e0} a_1) + \boldsymbol{I}_2 (\partial_{e2} a_0 - \partial_{e0} a_2) + \boldsymbol{I}_3 (\partial_{e3} a_0 - \partial_{e0} a_3) \\
& + \boldsymbol{I}_1 (\partial_{e3} a_2 - \partial_{e2} a_3) + \boldsymbol{I}_2 (\partial_{e1} a_3 - \partial_{e3} a_1) + \boldsymbol{I}_3 (\partial_{e2} a_1 - \partial_{e1} a_2)] \\
& + k_a [\boldsymbol{I}_0 (\partial_{g0} A_0 - \partial_{g1} A_1 - \partial_{g2} A_2 - \partial_{g3} A_3) \\
& + \boldsymbol{I}_1 (\partial_{g1} A_0 + \partial_{g0} A_1) + \boldsymbol{I}_2 (\partial_{g2} A_0 + \partial_{g0} A_2) + \boldsymbol{I}_3 (\partial_{g3} A_0 + \partial_{g0} A_3) \\
& + \boldsymbol{I}_1 (\partial_{g3} A_2 - \partial_{g2} A_3) + \boldsymbol{I}_2 (\partial_{g1} A_3 - \partial_{g3} A_1) + \boldsymbol{I}_3 (\partial_{g2} A_1 - \partial_{g1} A_2)] \\
& + k_a [-(\partial_{e0} A_0 + \partial_{e1} A_1 + \partial_{e2} A_2 + \partial_{e3} A_3) \\
& + \boldsymbol{i}_1 (\partial_{e0} A_1 - \partial_{e1} A_0) + \boldsymbol{i}_2 (\partial_{e0} A_2 - \partial_{e2} A_0) + \boldsymbol{i}_3 (\partial_{e0} A_3 - \partial_{e3} A_0) \\
& + \boldsymbol{i}_1 (\partial_{e3} A_2 - \partial_{e2} A_3) + \boldsymbol{i}_2 (\partial_{e1} A_3 - \partial_{e3} A_1) + \boldsymbol{i}_3 (\partial_{e2} A_1 - \partial_{e1} A_2)]
\end{aligned} \tag{11}$$

where, the first two terms are the field strength of the gravitational interaction, the last two terms are the field strength of the electromagnetic interaction. $\mathbb{B} = \mathbb{B}_g + k_b \mathbb{B}_e$; $\mathbb{B}_g = (b_1, b_2, b_3)$ and $\mathbb{B}_e = (B_1, B_2, B_3)$ are the field strength in G space and E space respectively; k_b is a coefficient. Selecting gauge equations $b_0 = 0$ and $B_0 = 0$ can simplify definition of field strength in the octonion space.

TABLE III: The octonion multiplication table.

	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{I}_0$	$\boldsymbol{I}_1$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$
1	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{I}_0$	$\boldsymbol{I}_1$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$
$\boldsymbol{i}_1$	$\boldsymbol{i}_1$	-1	$\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{I}_1$	$-\boldsymbol{I}_0$	$-\boldsymbol{I}_3$	$\boldsymbol{I}_2$
$\boldsymbol{i}_2$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_3$	-1	$\boldsymbol{i}_1$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$	$-\boldsymbol{I}_0$	$-\boldsymbol{I}_1$
$\boldsymbol{i}_3$	$\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_1$	-1	$\boldsymbol{I}_3$	$-\boldsymbol{I}_2$	$\boldsymbol{I}_1$	$-\boldsymbol{I}_0$
$\boldsymbol{I}_0$	$\boldsymbol{I}_0$	$-\boldsymbol{I}_1$	$-\boldsymbol{I}_2$	$-\boldsymbol{I}_3$	-1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$
$\boldsymbol{I}_1$	$\boldsymbol{I}_1$	$\boldsymbol{I}_0$	$-\boldsymbol{I}_3$	$\boldsymbol{I}_2$	$-\boldsymbol{i}_1$	-1	$-\boldsymbol{i}_3$	$\boldsymbol{i}_2$
$\boldsymbol{I}_2$	$\boldsymbol{I}_2$	$\boldsymbol{I}_3$	$\boldsymbol{I}_0$	$-\boldsymbol{I}_1$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_3$	-1	$-\boldsymbol{i}_1$
$\boldsymbol{I}_3$	$\boldsymbol{I}_3$	$-\boldsymbol{I}_2$	$\boldsymbol{I}_1$	$\boldsymbol{I}_0$	$-\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_1$	-1

B. Field source

The field source $\mathbb{S}(s_0, s_1, s_2, s_3, S_0, S_1, S_2, S_3)$ is defined as

$$\begin{aligned}
-\mu\mathbb{S} = \; & \lozenge^* \circ \mathbb{B} \\
= \; & [(\partial_{g0}b_0 + \partial_{g1}b_1 + \partial_{g2}b_2 + \partial_{g3}b_3) \\
& + \boldsymbol{i}_1(\partial_{g0}b_1 - \partial_{g1}b_0) + \boldsymbol{i}_2(\partial_{g0}b_2 - \partial_{g2}b_0) + \boldsymbol{i}_3(\partial_{g0}b_3 - \partial_{g3}b_0) \\
& + \boldsymbol{i}_1(\partial_{g3}b_2 - \partial_{g2}b_3) + \boldsymbol{i}_2(\partial_{g1}b_3 - \partial_{g3}b_1) + \boldsymbol{i}_3(\partial_{g2}b_1 - \partial_{g1}b_2)] \\
& + [-\boldsymbol{I}_0(\partial_{e0}b_0 + \partial_{e1}b_1 + \partial_{e2}b_2 + \partial_{e3}b_3) \\
& + \boldsymbol{I}_1(\partial_{e0}b_1 - \partial_{e1}b_0) + \boldsymbol{I}_2(\partial_{e0}b_2 - \partial_{e2}b_0) + \boldsymbol{I}_3(\partial_{e0}b_3 - \partial_{e3}b_0) \\
& + \boldsymbol{I}_1(\partial_{e2}b_3 - \partial_{e3}b_2) + \boldsymbol{I}_2(\partial_{e3}b_1 - \partial_{e1}b_3) + \boldsymbol{I}_3(\partial_{e1}b_2 - \partial_{e2}b_1)] \\
& + k_b[\boldsymbol{I}_0(\partial_{g0}B_0 + \partial_{g1}B_1 + \partial_{g2}B_2 + \partial_{g3}B_3) \\
& + \boldsymbol{I}_1(\partial_{g0}B_1 - \partial_{g1}B_0) + \boldsymbol{I}_2(\partial_{g0}B_2 - \partial_{g2}B_0) + \boldsymbol{I}_3(\partial_{g0}B_3 - \partial_{g3}B_0) \\
& + \boldsymbol{I}_1(\partial_{g2}B_3 - \partial_{g3}B_2) + \boldsymbol{I}_2(\partial_{g3}B_1 - \partial_{g1}B_3) + \boldsymbol{I}_3(\partial_{g1}B_2 - \partial_{g2}B_1)] \\
& + k_b[(\partial_{e0}B_0 + \partial_{e1}B_1 + \partial_{e2}B_2 + \partial_{e3}B_3) \\
& + \boldsymbol{i}_1(\partial_{e1}B_0 - \partial_{e0}B_1) + \boldsymbol{i}_2(\partial_{e2}B_0 - \partial_{e0}B_2) + \boldsymbol{i}_3(\partial_{e3}B_0 - \partial_{e0}B_3) \\
& + \boldsymbol{i}_1(\partial_{e2}B_3 - \partial_{e3}B_2) + \boldsymbol{i}_2(\partial_{e3}B_1 - \partial_{e1}B_3) + \boldsymbol{i}_3(\partial_{e1}B_2 - \partial_{e2}B_1)] \\
= \; & -\mu_g^g(s_0^g + \boldsymbol{i}_1 s_1^g + \boldsymbol{i}_2 s_2^g + \boldsymbol{i}_3 s_3^g) - \mu_g^e(\boldsymbol{I}_0 s_0^e + \boldsymbol{I}_1 s_1^e + \boldsymbol{I}_2 s_2^e + \boldsymbol{I}_3 s_3^e) \\
& - k_b\mu_e^g(S_0^g + \boldsymbol{i}_1 S_1^g + \boldsymbol{i}_2 S_2^g + \boldsymbol{i}_3 S_3^g) - k_b\mu_e^e(\boldsymbol{I}_0 S_0^e + \boldsymbol{I}_1 S_1^e + \boldsymbol{I}_2 S_2^e + \boldsymbol{I}_3 S_3^e)
\end{aligned} \tag{12}$$

where, the first two terms are the definition of momentum, and the last two terms are the definition of current; μ, μ_e^e, μ_e^g, μ_g^e and μ_g^g are coefficients.

The physical quantities in the quaternion space need to be extended to that in the octonion space. In the octonion space, the definition of physical quantity should be equal and consistent in the electromagnetic and gravitational interactions.

The velocity $\mathbb{V}(v_0, v_1, v_2, v_3, V_0, V_1, V_2, V_3)$ of particle is defined as

$$\mathbb{V} = \partial\mathbb{R}/\partial t = (v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3) + (\boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3) \tag{13}$$

where, the first term is the velocity in G space, and the second term is the velocity in E space.

In the octonion space, the charge and mass of particles are defined as follows

$$s_0^g + \boldsymbol{i}_1 s_1^g + \boldsymbol{i}_2 s_2^g + \boldsymbol{i}_3 s_3^g = Q_g^g(v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3) \tag{14a}$$
$$\boldsymbol{I}_0 s_0^e + \boldsymbol{I}_1 s_1^e + \boldsymbol{I}_2 s_2^e + \boldsymbol{I}_3 s_3^e = Q_g^e(\boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3) \tag{14b}$$
$$S_0^g + \boldsymbol{i}_1 S_1^g + \boldsymbol{i}_2 S_2^g + \boldsymbol{i}_3 S_3^g = Q_e^g(v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3) \tag{14c}$$
$$\boldsymbol{I}_0 S_0^e + \boldsymbol{I}_1 S_1^e + \boldsymbol{I}_2 S_2^e + \boldsymbol{I}_3 S_3^e = Q_e^e(\boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3) \tag{14d}$$

where, the first two equations are the definition of mass Q_g^g and Q_g^e respectively; the last two equations are the definition of charge Q_e^g and Q_e^e respectively. The charges Q_g^g, Q_g^e, Q_e^g and Q_e^e can be known as the 'general charge' uniformly.

In the octonion space, there are four types of field sources in the electromagnetic-gravitational field. There exist two parts of field sources in the electromagnetic interaction, one part lies in G space (1, $\boldsymbol{i}_1$, $\boldsymbol{i}_2$, $\boldsymbol{i}_3$) and the other in E space ($\boldsymbol{I}_0$, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$). So does the gravitational interaction.

C. Force and angular momentum

In the electromagnetic-gravitational field, the force can be defined by the linear momentum $\mathbb{P} = \mu\mathbb{S}/\mu_g^g$, field strength $\mathbb{B}$, and operator $\lozenge$. The field force $\mathbb{F}$ and variation rate $\mathbb{J}$ of field source are defined as

$$\mathbb{F} = \mathbb{B}^* \circ \mathbb{P} \ , \quad \mathbb{J} = \lozenge^* \circ \mathbb{P} \ . \tag{15}$$

The definition of variation rate $\mathbb{J}$ of field source is different from that of field force $\mathbb{F}$. But both of them have effects on the matter particle. The field force $\mathbb{F}$ includes the Lorentz force [8], while the variation rate $\mathbb{J}$ of field source includes the inertia force [9].

The angular momentum $\mathbb{M}$ is defined as (k_{rx} is a coefficient)

$$\mathbb{M} = (\mathbb{R} + k_{rx}\mathbb{X}) \circ \mathbb{P} \tag{16}$$

The $\mathbb{M}$ is defined by the linear momentum $\mathbb{P}$, field strength $\mathbb{B}$, radius vector $\mathbb{R}$, physical quantity $\mathbb{X}$ and operator $\lozenge$, and includes the angular momentum, work, kinetic energy, potential energy and force moment etc.

The work and moment derived from the field force are defined uniformly as

$$\mathbb{W}_1 = \mathbb{B} \circ (\mathbb{R} \circ \mathbb{P}) + \mathbb{B} \circ (k_{rx}\mathbb{X} \circ \mathbb{P}) \tag{17}$$

where, the physical quantity $\mathbb{B} \circ (\mathbb{X} \circ \mathbb{P})$ is a new part of the energy.

The kinetic energy, potential energy and moment derived from the variation rate of field source are defined as

$$\mathbb{W}_2 = \lozenge \circ (\mathbb{R} \circ \mathbb{P}) + \lozenge \circ (k_{rx}\mathbb{X} \circ \mathbb{P}) \tag{18}$$

where, the physical quantity $\lozenge \circ (\mathbb{X} \circ \mathbb{P})$ is the part of potential energy.

Both definitions of energy $\mathbb{W}_1$ and $\mathbb{W}_2$ should exclude the potential energy part $\mathbb{A} \circ \mathbb{P}$, if the definition of angular momentum is confined in $\mathbb{M} = \mathbb{R} \circ \mathbb{P}$. Hence this definition of angular momentum is inexact apparently. As shown in Aharonov-Bohm experiment [10], the field potential $\mathbb{A} = \lozenge \circ \mathbb{X}$ has effects on the energy $\lozenge \circ (\mathbb{X} \circ \mathbb{P})$, which includes the potential energy $\mathbb{A} \circ \mathbb{P}$. And the change of field potential can impact the movement of field source.

IV. EQUATIONS SET OF ELECTROMAGNETIC-GRAVITATIONAL FIELD

In the octonion electromagnetic-gravitational field, two types of the forces can be written together as follows from Eq.(15) ($\alpha = c$ is a coefficient)

$$\mathbb{Z} = \mathbb{F} + \alpha\mathbb{J} = (\mathbb{B} + \alpha\lozenge)^* \circ \mathbb{P} \tag{19}$$

And two types of energies can also be written as follows from Eqs.(17) and (18)

$$\mathbb{W} = \mathbb{W}_1 + \alpha\mathbb{W}_2 = (\mathbb{B} + \alpha\lozenge) \circ \mathbb{M} \tag{20}$$

The unified definitions of force Eq.(19) and energy Eq.(20) show that, previous equations set of the quaternion electromagnetic field and octonion electromagnetic-gravitational field should be supplemented and generalized. It also shows that the octonion differential operator $\lozenge$ needs to be generalized to a new operator $(\lozenge + \mathbb{B}/\alpha)$. The physical characteristics of electromagnetic-gravitational field can be described uniformly from many aspects.

A. Field source equation

The extended definition of force Eq.(19) shows that, the field source $\mathbb{S}$ needs to be revised and generalized to the form of following equation from Eq.(12)

$$\mu\mathbb{S} = -(\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{B} = (\mu\mathbb{S})' - \mathbb{B}^* \circ \mathbb{B}/\alpha \tag{21}$$

where, $(\mu\mathbb{S})' = -\lozenge^* \circ \mathbb{B}$ is the definition of the field source in Eq.(12) in the Maxwellian electromagnetic theory or the Newtonian gravitational theory.

As one part of field source $\mathbb{S}$, the term $(\mathbb{B}^* \circ \mathbb{B}/\alpha)$ is directly proportional of field energy density $(\mathbb{B}^* \circ \mathbb{B}/\mu_g^g)$. The force-balance equation can be obtained when the force $\mathbb{Z} = 0$ from Eq.(19). And the variation of term $(\mathbb{B}^* \circ \mathbb{B}/\alpha)$ will be one kind of necessary portion of force $\mathbb{Z}$.

TABLE IV: The subfield types of electromagnetic-gravitational field.

Operator	Gravitational Interaction	Electromagnetic Interaction
operator $\lozenge_g$	gravitational-gravitational subfield	electromagnetic-gravitational subfield
of G space	G mass, Q_g^g	G charge, Q_e^g
	intermediate particle, γ_g^g	intermediate particle, γ_e^g
operator $\lozenge_e$	gravitational-electromagnetic subfield	electromagnetic-electromagnetic subfield
of E space	E mass, Q_g^e	E charge, Q_e^e
	intermediate particle, γ_g^e	intermediate particle, γ_e^e

B. Power equation

The extended definition of energy Eq.(20) shows that, the power $\mathbb{N}$ needs to be revised and generalized to the form of the following equation.

$$\mathbb{N} = (\mathbb{B} + \alpha\lozenge)^* \circ \mathbb{W} \tag{22}$$

In the electromagnetic-gravitational field, the physical quantity $\mathbb{X}$ has effect on field potential $\lozenge \circ \mathbb{X}$, angular momentum $\mathbb{X} \circ \mathbb{P}$, energy $\mathbb{B} \circ (\mathbb{X} \circ \mathbb{P})$ and power $(\mathbb{B} \circ \mathbb{B}^*) \circ (\mathbb{X} \circ \mathbb{P})$ etc. The introduction of physical quantity $\mathbb{X}$ makes the definition of angular momentum $\mathbb{M}$ and energy $\mathbb{W}$ more integrated, and the theory more self-consistent.

In Eq.(20), the conservation of angular momentum in electromagnetic-gravitational field can be gained when $\mathbb{W} = 0$. And the energy conservation of electromagnetic-gravitational field can be attained from Eq.(22) when $\mathbb{N} = 0$.

C. Dark matter

In the octonion spacetime, there exist four types of subfields and their field source of electromagnetic-gravitational field. In the Table 4, the electromagnetic-gravitational (E-G) subfield and gravitational-gravitational (G-G) subfield are 'electromagnetic field' and 'modified gravitational filed' respectively. Their general charges are G charge and G mass respectively. The electromagnetic-electromagnetic (E-E) subfield and gravitational-electromagnetic (G-E) subfield are both long range fields and candidates of the 'dark matter field'. Their general charges (E charge and E mass) are candidates of 'dark matter'. The physical features of the dark matter meet the requirement of Eqs.(10), (11), (16), (19), (20), (21) and (22) .

In the general charge (Q_g^g, Q_g^e, Q_e^g and Q_e^e) and intermediate particle (γ_g^g, γ_g^e, γ_e^g and γ_e^e), two types of general charges (familiar charge Q_e^g and mass Q_g^g) and one type of intermediate particle (photon γ_e^g) have been found. Hence the remaining two types of general charges and three types of intermediate particles are left to be found in Table 4. From Eq.(12), we find that the electromagnetic-gravitational field can transform one form of intermediate particle into another. In other words, the photon γ_e^g can be transformed into the γ_g^g, γ_g^e, or γ_e^e .

The particles of ordinary matter (electron and proton etc.) possess the G charge together with G mass. The particles of dark matter may possess the E charge with E mass, or G mass with E charge, etc. It can be predicted that field strength of electromagnetic-electromagnetic and gravitational-electromagnetic subfields must be very weak and a little less than that of the gravitational-gravitational subfield. Otherwise they should been detected for a long time. Hence the field strength of electromagnetic-electromagnetic and gravitational-electromagnetic subfields may be equal, and a little less than that of gravitational-gravitational subfield.

TABLE V: The comparison between ordinary matter with dark matter.

	Ordinary Matter		Dark Matter	
subfield	G-G subfield	E-G subfield	G-E subfield	E-E subfield
field potential	$\lozenge_g \circ \mathbb{X}_g$	$\lozenge_g \circ \mathbb{X}_e$	$\lozenge_e \circ \mathbb{X}_g$	$\lozenge_e \circ \mathbb{X}_e$
field strength	$\lozenge_g \circ \mathbb{A}_g$	$\lozenge_g \circ \mathbb{A}_e$	$\lozenge_e \circ \mathbb{A}_g$	$\lozenge_e \circ \mathbb{A}_e$
field source	$\lozenge_g^* \circ \mathbb{B}_g$	$\lozenge_g^* \circ \mathbb{B}_e$	$\lozenge_e^* \circ \mathbb{B}_g$	$\lozenge_e^* \circ \mathbb{B}_e$

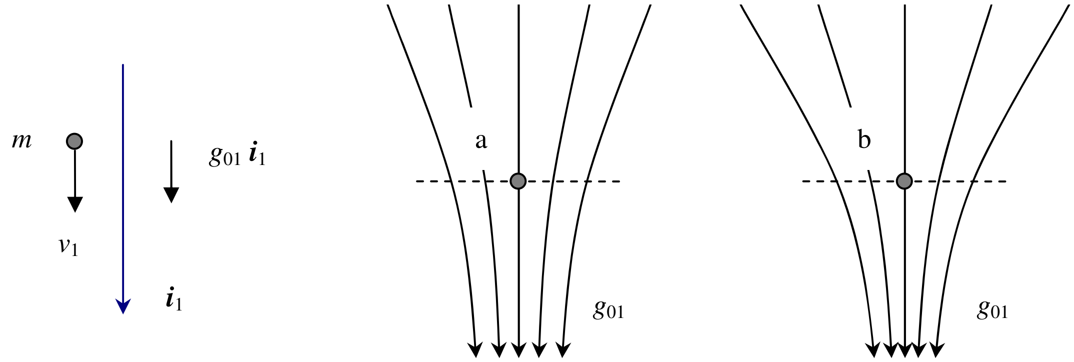

FIG. 2: Linear movement. The change of spatial distribution of mass m will affect free-fall movement of particle m. The linear movement status of particle m in point a and b are different, though the field strength are equivalent in those points.

V. EXPERIMENTS AND PHENOMENA

The field sources (current and momentum) in Eq.(19) stays in the force-balance status in the electromagnetic-gravitational field when $\mathbb{Z} = 0$. The influence between the 'dark matter field' (electromagnetic-electromagnetic and gravitational-electromagnetic subfields) and the 'modified gravitational field' (gravitational-gravitational subfield) on the force $\mathbb{Z}$ can be discussed with this feature. The conservation of angular momentum in Eq.(20) in the electromagnetic-gravitational field can be achieved when $\mathbb{W} = 0$. The influence between the 'modified gravitational field' and 'dark matter field' on the angular momentum and energy can be considered by this property. It is simpler and more convenient to represent the field source $(\mu\mathbb{S})'$ as $\mu\mathbb{S}$ in the following discussion.

A. Linear movement

In the gravitational-gravitational subfield $(g_{01}, 0, 0)$ in Fig.2, the particle m is in free-fall movement and its momentum is $(s^g_0, s^g_1, 0, 0)$. When the particle m is in weightlessness status,

$$\mathbb{Z} = (\boldsymbol{i}_1 g_{01} + \alpha\lozenge)^* \circ \left\{\mu^g_g(s^g_0 + \boldsymbol{i}_1 s^g_1) - g^2_{01}/\alpha\right\}/\mu^g_g = 0 \tag{23}$$

then eight equations can be obtained from Eq.(23). And its equation in $\boldsymbol{i}_1$ direction is

$$\mu^g_g g_{01} s^g_0 + \alpha\mu^g_g(\partial_{g1} s^g_0 - \partial_{g0} s^g_1) - (\partial_{g1} g^2_{01} + g^3_{01}/\alpha) = 0 \tag{24}$$

When the last term of the above equation is zero, we can achieve

$$g_{01} s^g_0 = c(\partial_{g0} s^g_1 - \partial_{g1} s^g_0) \tag{25}$$

where, the physical quantity can be written in familiar form, $g_{01} = g/c$, $s^g_0 = mc$, $s^g_1 = mv_1$.

The above shows that, when the spatial distribution of the mass m changes, as one part of inertia force, $\partial_{g1} s^g_0$ will affect free-fall movement of particle m. The omitted term $(\partial_{g1} g^2_{01} + g^3_{01}/\alpha)$ in Eq.(24) will influence free-fall movement of particle m also. Then the linear movement status of particle m in the point a and b are different in Fig.2, though the field strength g_{01} in the point a and b are equal.

B. Stationary status

In the area, where coexist the electromagnetic-gravitational subfield $(b_{01}, 0, 0)$ and gravitational-gravitational subfield $(g_{01}, 0, 0)$. The electric charged particle q is in the stationary status in the area in Fig.3, its momentum is $(s^g_0, 0, 0, 0)$ and its current is $(S^g_0, 0, 0, 0)$. When the particle q stays in the force-balance status

$$\mathbb{Z} = \left\{(\boldsymbol{i}_1 g_{01} + k_a \boldsymbol{I}_1 b_{01}) + \alpha\lozenge\right\}^* \circ \left\{(\mu^g_g s^g_0 + k_b \mu^g_e \boldsymbol{I}_0 S^g_0) - (g^2_{01} + k^2_a b^2_{01})/\alpha\right\}/\mu^g_g = 0 \tag{26}$$

so its equation in $\boldsymbol{i}_1$ direction is

$$(\mu^g_g g_{01} s^g_0 - k_b \mu^g_e k_a b_{01} S^g_0) + \alpha(\mu^g_g \partial_{g1} s^g_0 - k_b \mu^g_e \partial_{e1} S^g_0) - \partial_{g1}(g^2_{01} + k^2_a b^2_{01}) - g_{01}(g^2_{01} + k^2_a b^2_{01})/\alpha = 0 \tag{27}$$

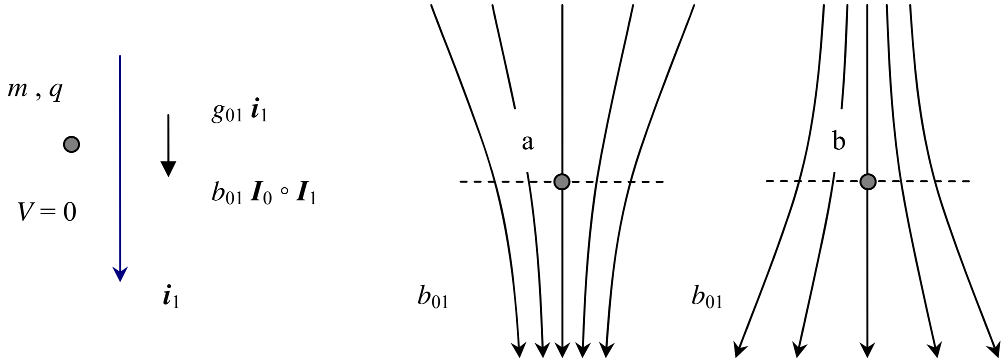

FIG. 3: Stationary status. The transform of energy density in the electromagnetic-gravitational field will affect the force-balance of charged particle including the stationary status of electric charged particle. In consequence the stationary status of electric charged particle q in point a and b will be different.

The last two terms of the above equation show that, the transform of energy density in the electromagnetic-gravitational and the gravitational-gravitational subfields will affect the force-balance of charged particle. When the sum of above last three terms equals to zero, and $k_a k_b = \mu_g^g / \mu_e^g$, we can conclude from above

$$g_{01} s_0^g - b_{01} S_0^g = 0$$

where, the physical quantity can be written in the familiar form, $b_{01} = E/c, g_{01} = g/c, s_0^g = mc, S_0^g = qc$.

The last three terms in Eq.(27) will impact the stationary status of electric charged particle. So the stationary status of electric charged particle q in point a and b will be different in Fig.3, though the field strength b_{01} in point a and b are equal.

C. Planar circumference movement of dark matter

In the electromagnetic-gravitational subfield (0, 0, b_{12}), the charged particle q is in the planar circumference movement in Fig.4. Its momentum is $(s_0^g, s_1^g, s_2^g, 0)$, while its two types of currents are $(S_0^e, S_1^e, S_2^e, 0)$ and $(S_0^g, S_1^g, S_2^g, 0)$ respectively. When the particle q is in the force-balance status,

$$\begin{aligned} \mathbb{Z} = \quad & (\boldsymbol{I}_3 k_a b_{12} + \alpha \Diamond)^* \circ \\ & \left\{ \mu_g^g (s_0^g + \boldsymbol{i}_1 s_1^g + \boldsymbol{i}_2 s_2^g) + k_b \mu_e^e (\boldsymbol{I}_0 S_0^e + \boldsymbol{I}_1 S_1^e + \boldsymbol{I}_2 S_2^e) + k_b \mu_e^g (S_0^g + \boldsymbol{i}_1 S_1^g + \boldsymbol{i}_2 S_2^g) - k_a^2 b_{12}^2 / \alpha \right\} / \mu_g^g = 0 \end{aligned} \tag{28}$$

so its equations in $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ directions are respectively

$$\begin{aligned} & k_a k_b b_{12} \mu_e^e S_2^e + \alpha \mu_g^g (-\partial_{g0} s_1^g + \partial_{g1} s_0^g - \partial_{g3} s_2^g) - \partial_{g1} (k_a b_{12})^2 \\ & + \alpha k_b \mu_e^e (\partial_{e0} S_1^e - \partial_{e1} S_0^e + \partial_{e3} S_2^e) + \alpha k_b \mu_e^g (-\partial_{g0} S_1^g - \partial_{g1} S_0^g + \partial_{g3} S_2^g) = 0 \end{aligned} \tag{29a}$$

$$\begin{aligned} & -k_a k_b b_{12} \mu_e^e S_1^e + \alpha \mu_g^g (-\partial_{g0} s_2^g + \partial_{g2} s_0^g + \partial_{g3} s_1^g) - \partial_{g2} (k_a b_{12})^2 \\ & + \alpha k_b \mu_e^e (\partial_{e0} S_2^e - \partial_{e2} S_0^e - \partial_{e3} S_1^e) + \alpha k_b \mu_e^g (-\partial_{g0} S_2^g + \partial_{g2} S_0^g + \partial_{g3} S_1^g) = 0 \end{aligned} \tag{29b}$$

It makes out that field source $(S_0^e, S_1^e, S_2^e, 0)$ of 'dark matter field' has direct influence on the movement of field sources $(S_0^g, S_1^g, S_2^g, 0)$ of familiar 'electromagnetic field'.

When the sum of last three terms in above each equation equals to 0, and $k_a k_b = \mu_g^g / \mu_e^g$, the force-balance equations of charged particle, which are in the circumference movement in electromagnetic field and modified gravitational field, can be achieved from above

$$(\mu_g^g \mu_e^e / \mu_e^g) b_{12} S_2^e + c(-\partial_{g0} s_1^g + \partial_{g1} s_0^g - \partial_{g3} s_2^g) = 0 \tag{30a}$$

$$(\mu_g^g \mu_e^e / \mu_e^g) b_{12} S_1^e - c(-\partial_{g0} s_2^g + \partial_{g2} s_0^g + \partial_{g3} s_1^g) = 0 \tag{30b}$$

The above equations show that, as the parts of inertia force, the terms $(\partial_{g1} s_0^g - \partial_{g3} s_2^g)$ and $(\partial_{g2} s_0^g + \partial_{g3} s_1^g)$ will affect the planar circumference movement of charged particle. The terms $\partial_{g1} b_{12}^2$ in Eq.(29a) and $\partial_{g2} b_{12}^2$ in Eq.(29b) will affect the planar circumference movement of charged particle q also.

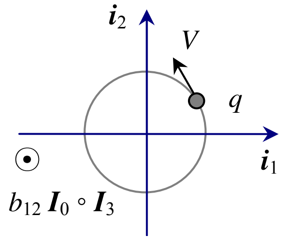

FIG. 4: Planar circumference movement. The field source of 'dark matter field' has direct influence on the movement of field source of familiar 'electromagnetic field'.

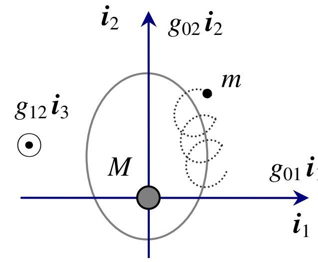

FIG. 5: Near circularity and rotation. The omitted 'magnetic' part of gravitational-gravitational subfield strength affects the movement of particle, and is one of important factors resulting in rotation of celestial bodies.

D. Near circularity and rotation

The large-mass object M generates the field strength (g_{01}, g_{02}, g_{12}) of the gravitational-gravitational subfield in its equator plane. The particle m moves around M in planar helix, and its momentum in orbit plane is $(s_0^g, s_1^g, s_2^g, 0)$. The orbit plane of particle m is superposed with the equator plane of large-mass object M in Fig.5. When particle m stays in force-balance status,

$$\mathbb{Z} = \quad (\boldsymbol{i}_1 g_{01} + \boldsymbol{i}_2 g_{02} + \boldsymbol{i}_3 g_{12} + \alpha \lozenge)^* \circ \left\{ \mu_g^g (s_0^g + \boldsymbol{i}_1 s_1^g + \boldsymbol{i}_2 s_2^g) - B^2/\alpha \right\} / \mu_g^g = 0 \tag{31}$$

so its equations in $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ directions are

$$\mu_g^g (g_{01} s_0^g + g_{12} s_2^g) + \alpha \mu_g^g (-\partial_{g0} s_1^g + \partial_{g1} s_0^g - \partial_{g3} s_2^g) - \partial_{g1} B^2 - g_{01} B^2/\alpha = 0 \tag{32a}$$

$$\mu_g^g (g_{02} s_0^g - g_{12} s_1^g) + \alpha \mu_g^g (-\partial_{g0} s_2^g + \partial_{g2} s_0^g + \partial_{g3} s_1^g) - \partial_{g2} B^2 - g_{02} B^2/\alpha = 0 \tag{32b}$$

where, $B^2 = g_{01}^2 + g_{02}^2 + g_{12}^2$.

When the sum of last two terms in above each equation is equal approximately to zero, we can attain

$$(g_{01} s_0^g + g_{12} s_2^g) + c(-\partial_{g0} s_1^g + \partial_{g1} s_0^g - \partial_{g3} s_2^g) = 0 \tag{33a}$$

$$(g_{02} s_0^g - g_{12} s_1^g) + c(-\partial_{g0} s_2^g + \partial_{g2} s_0^g + \partial_{g3} s_1^g) = 0 \tag{33b}$$

When $g_{12} = 0$, we can learn that the particle m moves around object M in planar ellipse from above equations. That is the near circularity of planets. When $g_{01} = g_{02} = 0$, it can be found that the particle m moves in planar circumference from above equations. That is the rotation of planets. Therefore, in gravitational-gravitational subfield (g_{01}, g_{02}, g_{12}) , the centroid movement of particle m is the superposition of these two types of movements. That is, the center of planar circumference movement of particle moves in planar ellipse. When the particle m moves around big mass M, the movement of particle m is in revolution, rotation and swing because of weakness and nonuniform distribution of g_{12} [11]. It illustrates that, the omitted 'magnetic' part g_{12} of gravitational-gravitational subfield strength affects the movement of planet m, and is one of important factors resulting in rotation of celestial bodies.

As shown above, the terms $(\partial_{g1} s_0^g - \partial_{g3} s_2^g)$ and $(\partial_{g2} s_0^g + \partial_{g3} s_1^g)$, as certain parts of inertia force, will affect the planar helix movement of particle in Fig.6. In the solar system, when spatial distribution of planetary mass and field energy density are invariable, or when the planetary momentum is invariable along the direction perpendicular to the Ecliptic Plane, the movement of planet is consistent with the Newtonian gravitational theory. Contrarily, in the Milky Way Galaxy, when the spatial distribution of solar system mass and field energy density are variable, or when the angular momentum of the solar system is variable along the direction perpendicular to the Galactic Equator Plane, the centrifugal force of solar system will change, and then the movement of solar system will betray the familiar gravitational theory. If the change of mass, energy density and momentum lead to counteract the certain parts of centrifugal force, it will result in the revolution speed increase to generate enough centrifugal force to balance gravitational interaction of the Milky Way Galaxy. Therefore the inferences may solve inconsistency between the overspeed rotation of galaxy and deficiency of gravitation in some extent [12].

E. Near coplanarity and corevolving

On the orbit plane of particle m which momentum is $(s_0^g, s_1^g, s_2^g, s_3^g)$, the gravitational-gravitational subfield $(g_{01} + g_{23}, g_{02} + g_{31}, g_{03} + g_{12})$ and the electromagnetic-gravitational subfield $(b_{01} + b_{23}, b_{02} + b_{31}, b_{03} + b_{12})$ are generated by

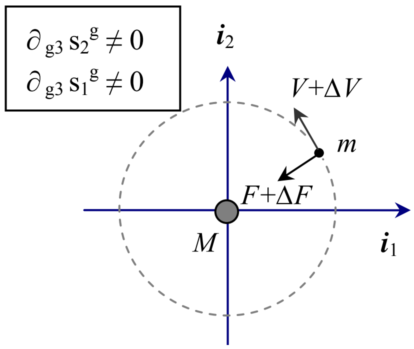

FIG. 6: Overspeed revolution. If the change of mass, energy density and momentum lead to counteract the certain parts of centrifugal force, it will result in the revolution speed increase to generate enough centrifugal force to balance gravitational interaction.

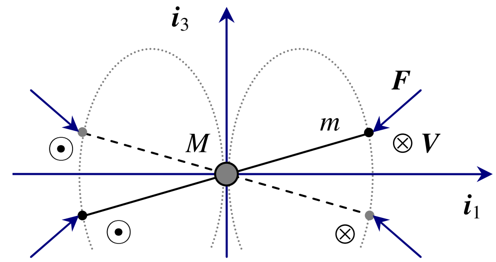

FIG. 7: Near coplanarity and corevolving. When the sense of revolution of particle m is in the same sense of rotation of object M, the movement of particle m is attracted by the force which points to the orbit plane, hence the movement of particle m is stable.

the large-mass object M which owns rotation and charge in Fig.7. When the particle m, which moves around M in spatial helix, is in the force-balance status,

$$\begin{aligned}\mathbb{Z} = \ & [\boldsymbol{i}_1(g_{01}+g_{23}) + \boldsymbol{i}_2(g_{02}+g_{31}) + \boldsymbol{i}_3(g_{03}+g_{12}) \\ & + k_a \{ \boldsymbol{I}_1(b_{01}+b_{23}) + \boldsymbol{I}_2(b_{02}+b_{31}) + \boldsymbol{I}_3(b_{03}+b_{12})\} + \alpha\lozenge]^* \circ \\ & \left\{\mu_g^g(s_0^g + \boldsymbol{i}_1 s_1^g + \boldsymbol{i}_2 s_2^g + \boldsymbol{i}_3 s_3^g) - B^2/\alpha\right\}/\mu_g^g = 0 \end{aligned} \tag{34}$$

so its equations in $\boldsymbol{i}_1$, $\boldsymbol{i}_2$ and $\boldsymbol{i}_3$ directions respectively are,

$$\begin{aligned}& \mu_g^g \{(g_{01}+g_{23})s_0^g + (g_{02}+g_{31})s_3^g - (g_{03}+g_{12})s_2^g\} \\ & + \alpha\mu_g^g(-\partial_{g0}s_1^g + \partial_{g1}s_0^g + \partial_{g2}s_3^g - \partial_{g3}s_2^g) - \partial_{g1}B^2 - (g_{01}+g_{23})B^2/\alpha = 0 \end{aligned} \tag{35a}$$

$$\begin{aligned}& \mu_g^g \{(g_{02}+g_{31})s_0^g - (g_{01}+g_{23})s_3^g + (g_{03}+g_{12})s_1^g\} \\ & + \alpha\mu_g^g(-\partial_{g0}s_2^g - \partial_{g1}s_3^g + \partial_{g2}s_0^g + \partial_{g3}s_1^g) - \partial_{g2}B^2 - (g_{02}+g_{31})B^2/\alpha = 0 \end{aligned} \tag{35b}$$

$$\begin{aligned}& \mu_g^g \{(g_{03}+g_{12})s_0^g + (g_{01}+g_{23})s_2^g - (g_{02}+g_{31})s_1^g\} \\ & + \alpha\mu_g^g(-\partial_{g0}s_3^g + \partial_{g1}s_2^g - \partial_{g2}s_1^g + \partial_{g3}s_0^g) - \partial_{g3}B^2 - (g_{03}+g_{12})B^2/\alpha = 0 \end{aligned} \tag{35c}$$

where, $B^2 = (g_{01}+g_{23})^2 + (g_{02}+g_{31})^2 + (g_{03}+g_{12})^2 + k_a^2\left\{(b_{01}+b_{23})^2 + (b_{02}+b_{31})^2 + (b_{03}+b_{12})^2\right\}$.

The last two terms of above equations show that, the change of energy density (B^2/μ_g^g) in the gravitational-gravitational and electromagnetic-gravitational subfields will influence the movement of particle m. When the sum of last two terms in above each equation is zero, the equations of gravitational-gravitational subfield can be summarized as follows

$$\{(g_{01}+g_{23})s_0^g + (g_{02}+g_{31})s_3^g - (g_{03}+g_{12})s_2^g\} + c(-\partial_{g0}s_1^g + \partial_{g1}s_0^g + \partial_{g2}s_3^g - \partial_{g3}s_2^g) = 0 \tag{36a}$$

$$\{(g_{02}+g_{31})s_0^g - (g_{01}+g_{23})s_3^g + (g_{03}+g_{12})s_1^g\} + c(-\partial_{g0}s_2^g + \partial_{g2}s_0^g + \partial_{g3}s_1^g - \partial_{g1}s_3^g) = 0 \tag{36b}$$

$$\{(g_{03}+g_{12})s_0^g + (g_{01}+g_{23})s_2^g - (g_{02}+g_{31})s_1^g\} + c(-\partial_{g0}s_3^g + \partial_{g3}s_0^g + \partial_{g1}s_2^g - \partial_{g2}s_1^g) = 0 \tag{36c}$$

From above equations we can learn that, when the sense of revolution of particle m is in the same sense of rotation of object M, the movement of particle m is attracted by the force which points to the orbit plane, hence the movement of particle m is stable. That is the near coplanarity of planets. The particle m, which moves around object M in spatial helix, revolves and waves up and down slowly around the equator plane of object M (the earth's movement relatives to the equator plane of the sun; the movement of solar system and pulsar relative to the galactic plane of the Milky Way Galaxy), at the same time it rotates with swing [13]. When the sense of revolution of particle m is in opposite sense of rotation of object M, the movement of particle m is repelled by the force which deviates from the orbit plane and hence the movement of particle m is unstable. It shows that, the sense of revolution of particle m affects directly its movement stability. That is corevolving (or prograde) of planets. Therefore, the solar system and galaxies are inclined to thin disk structure as a whole [14].

To be extended, two galaxies rotating in the same sense of rotation are in attraction and revolution around each other, even shorten their interval and swallow up each other to become one galaxy. Two galaxies rotating in opposite

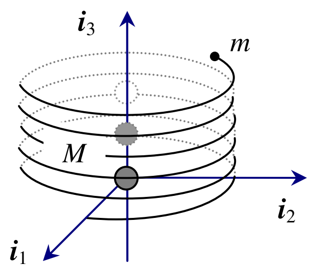

FIG. 8: Spatial helix movement. In Eq.(39), when the right side of equal mark in last equation is equal to zero, and the first three equations are not all zero, the angular momentum M_3^g is close to be in conservation and in consequence more complicated situations will appear on the revolution orbit of particle m.

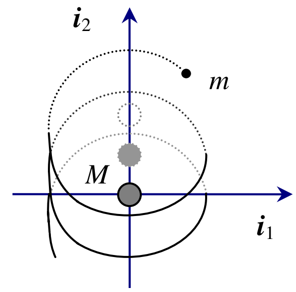

FIG. 9: Planar helix movement of celestial body. It shows that there are the coupling influences between the 'dark matter field' and 'modified gravitational field'. When the last term is zero in Eq.(42), there will be no influence of the 'dark matter field' upon the 'modified gravitational field'.

senses of rotation will repel and increase their interval. Two galaxies rotating in arbitrary senses of rotation will be affected by more complicated forces and moment [15].

F. Spatial helix movement

On the orbit plane of particle m, which angular momentum is $(M_0^g, M_1^g, M_2^g, M_3^g)$, the gravitational-gravitational subfield $(g_{01}+g_{23}, g_{02}+g_{31}, g_{03}+g_{12})$ is generated by large-mass object M which maintain its rotation simultaneously in Fig.8. When the particle m, which moves around M in spatial helix, stays in conservation of angular momentum,

$$\mathbb{W} = \{\boldsymbol{i}_1(g_{01}+g_{23}) + \boldsymbol{i}_2(g_{02}+g_{31}) + \boldsymbol{i}_3(g_{03}+g_{12}) + \alpha\lozenge\} \circ (M_0^g + \boldsymbol{i}_1 M_1^g + \boldsymbol{i}_2 M_2^g + \boldsymbol{i}_3 M_3^g) = 0 \tag{37}$$

so its equations in $1, \boldsymbol{i}_1, \boldsymbol{i}_2$ and $\boldsymbol{i}_3$ directions are respectively

$$c(\partial_{g0}M_0^g - \partial_{g1}M_1^g - \partial_{g2}M_2^g - \partial_{g3}M_3^g) - \{(g_{01}+g_{23})M_1^g + (g_{02}+g_{31})M_2^g + (g_{03}+g_{12})M_3^g\} = 0 \tag{38a}$$
$$c(\partial_{g0}M_1^g + \partial_{g1}M_0^g + \partial_{g2}M_3^g - \partial_{g3}M_2^g) + \{(g_{01}+g_{23})M_0^g + (g_{02}+g_{31})M_3^g - (g_{03}+g_{12})M_2^g\} = 0 \tag{38b}$$
$$c(\partial_{g0}M_2^g - \partial_{g1}M_3^g + \partial_{g2}M_0^g + \partial_{g3}M_1^g) + \{(g_{02}+g_{31})M_0^g - (g_{01}+g_{23})M_3^g + (g_{03}+g_{12})M_1^g\} = 0 \tag{38c}$$
$$c(\partial_{g0}M_3^g + \partial_{g1}M_2^g - \partial_{g2}M_1^g + \partial_{g3}M_0^g) + \{(g_{03}+g_{12})M_0^g + (g_{01}+g_{23})M_2^g - (g_{02}+g_{31})M_1^g\} = 0 \tag{38d}$$

In the gravitational-gravitational subfield, above equations set show that, the conservation of angular momentum will be affected by the field strength, angular momentum and its variation rate etc. When $M_1^g = M_2^g = 0$, we can obtain from the above equations set

$$\partial_{g3}M_3^g = \partial_{g0}M_0^g - \{(g_{03}+g_{12})M_3^g\}/c \tag{39a}$$
$$\partial_{g2}M_3^g = -\partial_{g1}M_0^g - \{(g_{01}+g_{23})M_0^g - (g_{02}+g_{31})M_3^g\}/c \tag{39b}$$
$$\partial_{g1}M_3^g = \partial_{g2}M_0^g + \{(g_{02}+g_{31})M_0^g - (g_{01}+g_{23})M_3^g\}/c \tag{39c}$$
$$\partial_{g0}M_3^g = -\partial_{g3}M_0^g - \{(g_{03}+g_{12})M_0^g\}/c \tag{39d}$$

When all terms in the right side of equal marks of the above equations set are zero, the angular momentum M_3^g of particle m along the sense of revolution is in conservation. And if the right side of equal mark of the last equation is equal to zero approximately, whilst the right side of equal marks of the first three equations are not all zero, the angular momentum M_3^g is close to be in conservation and consequentially more complicated situations will appear on the revolution orbit of particle m. For example, the asteroid will probably come near to the planet gradually, and the moon may perhaps keep away from the earth in the solar system [16, 17].

G. Planar helix movement of dark matter

The field strength of gravitational-gravitational and gravitational-electromagnetic subfields in Fig.9, which in equator plane of large-mass object M, are (g_{01}, g_{02}, g_{12}) and (G_{01}, G_{02}, G_{12}) respectively. The particle m moves around

M in planar helix, and its two types of momentums in orbit plane are $(s_0^g, s_1^g, s_2^g, 0)$ and $(s_0^e, s_1^e, s_2^e, 0)$ respectively. The orbit plane of particle m is superposed with equator plane of large-mass object M. When the particle m is in force-balance status,

$$\begin{aligned}\mathbb{Z} = \quad & (\boldsymbol{i}_1 g_{01} + \boldsymbol{i}_2 g_{02} + \boldsymbol{i}_3 g_{12} + \boldsymbol{I}_1 G_{01} + \boldsymbol{I}_2 G_{02} + \boldsymbol{I}_3 G_{12} + \alpha \lozenge)^* \circ \\ & \left\{ \mu_g^g (s_0^g + \boldsymbol{i}_1 s_1^g + \boldsymbol{i}_2 s_2^g) + \mu_g^e (\boldsymbol{I}_0 s_0^e + \boldsymbol{I}_1 s_1^e + \boldsymbol{I}_2 s_2^e) - B^2/\alpha \right\} / \mu_g^g = 0 \end{aligned} \tag{40}$$

so its equations in $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ directions are respectively

$$\begin{aligned}& \mu_g^g (g_{01} s_0^g - g_{12} s_2^g) + \alpha \mu_g^g (-\partial_{g0} s_1^g + \partial_{g1} s_0^g - \partial_{g3} s_2^g) \\ & + \mu_g^e \left\{ \alpha (\partial_{e0} s_1^e - \partial_{e1} s_0^e + \partial_{e3} s_2^e) + (G_{12} s_2^e - G_{01} s_0^e) \right\} - (g_{01} B^2/\alpha + \partial_{g1} B^2) = 0 \end{aligned} \tag{41a}$$

$$\begin{aligned}& \mu_g^g (g_{02} s_0^g + g_{12} s_1^g) + \alpha \mu_g^g (-\partial_{g0} s_2^g + \partial_{g2} s_0^g + \partial_{g3} s_1^g) \\ & + \mu_g^e \left\{ \alpha (\partial_{e0} s_2^e - \partial_{e2} s_0^e - \partial_{e3} s_1^e) - (G_{12} s_1^e + G_{02} s_0^e) \right\} - (g_{02} B^2/\alpha + \partial_{g2} B^2) = 0 \end{aligned} \tag{41b}$$

where, $B^2 = g_{01}^2 + g_{02}^2 + g_{12}^2 + G_{01}^2 + G_{02}^2 + G_{12}^2$.

When the last term of above each equation is zero approximately, we can conclude

$$\begin{aligned}& (g_{01} s_0^g - g_{12} s_2^g) + c(-\partial_{g0} s_1^g + \partial_{g1} s_0^g - \partial_{g3} s_2^g) \\ & + (\mu_g^e / \mu_g^g) \left\{ c(\partial_{e0} s_1^e - \partial_{e1} s_0^e + \partial_{e3} s_2^e) + (G_{12} s_2^e - G_{01} s_0^e) \right\} = 0 \end{aligned} \tag{42a}$$

$$\begin{aligned}& (g_{02} s_0^g + g_{12} s_1^g) + c(-\partial_{g0} s_2^g + \partial_{g2} s_0^g + \partial_{g3} s_1^g) \\ & + (\mu_g^e / \mu_g^g) \left\{ c(\partial_{e0} s_2^e - \partial_{e2} s_0^e - \partial_{e3} s_1^e) - (G_{12} s_1^e + G_{02} s_0^e) \right\} = 0 \end{aligned} \tag{42b}$$

In above equations set, the above equations are the movement equations of gravitational-gravitational subfield, including the affecting from the gravitational-electromagnetic subfield. It figures out that there are some coupling influences between the 'dark matter field' (gravitational-electromagnetic subfield) and 'modified gravitational field' (gravitational-gravitational subfield). When the last term is zero in the above each equation, there will be no influence of the 'dark matter field' (gravitational-electromagnetic subfield) upon the 'modified gravitational field' (gravitational-gravitational subfield).

If the preceding field strength and field source are extended to four types of field strength and field source of electromagnetic-gravitational field respectively, many extra and also more complicated equations set of interplay between 'dark matter field' and 'modified gravitational field', will be gained from Eqs.(19) and (20).

VI. CONCLUSIONS

In the observed phenomena of the celestial body, which are inconsistent with the results of current gravitational theory, some are caused by the modified gravitational field (gravitational-gravitational subfield) in the electromagnetic-gravitational field, and some are caused by the dark matter field (electromagnetic-electromagnetic and gravitational-electromagnetic subfields).

The gravitational-gravitational subfield is the modified gravitational field, which includes familiar Newtonian gravitational field. The features of modified gravitational field predict that, (1) the planetary orbits possess near coplanarity, near circularity and corevolving, and the planets own rotation property, (2) the centrifugal force of celestial body will change and lead to fluctuation of revolution speed, when the field energy density or the angular momentum of celestial body along the sense of revolution varied, (3) the two galaxies rotating in the same sense of rotation will attract and revolve around each other, hence shorten their interval and swallow up each other to become one galaxy.

The electromagnetic-electromagnetic and gravitational-electromagnetic subfields are both long range fields and candidates of dark matter field. Their general charges are candidates of dark matter. The combination of above general charges and the general charges of gravitational-gravitational or electromagnetic-gravitational subfields are candidates of dark matter also. The field strength of electromagnetic-electromagnetic and gravitational-electromagnetic subfields may be equal and very weak, and a little less than that of gravitational-gravitational subfield. The features of dark matter predict that, (1) the dark matter field possesses two sorts of general charges makes the dark matter particles very diversiform, (2) the field strength of two types of dark matter fields are both weaker than that of the Newtonian gravitational filed, (3) the distribution of dark matter will affect the stabilizing velocity of celestial body. If the field sources of the electromagnetic-electromagnetic and gravitational-electromagnetic subfields distribute properly, the obtained gravity of celestial body will increase, and the stabilizing velocity of celestial body will also increase correspondingly.

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